Indian Institute of Information Technology Allahabad
Discrete Mathematical Structures (DMS)
Tentative Marking Scheme

Program: B.Tech. 2^{nd} Semester (IT) Full Marks: 12
Date: Feb 02, 2026 Time: 06:30 PM - 07:00 PM

Instructions: Attempt all the questions. All the notations are standard and same as used in the lecture notes. No marks will be given if proper justification is not provided.

1. By using laws of propositions, show that $\neg p(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. In each step, write the explicit name of the applicable law. [4]

   Solution: $\neg(p \vee (\neg p \wedge q))$

   $\equiv \neg p \wedge \neg(\neg p \wedge q)$ (by De Morgan's Law)

   $\equiv \neg p \wedge (p \vee \neg q)$ (by De Morgan's Law and Double negation Law)

   $\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ (by Distributive Law)

   $\equiv F \vee (\neg p \wedge \neg q)$ (by Inverse Law)

   $(\neg p \wedge \neg q)$. (by Identity Law)

2. We say that a function $f : \mathbb{N} \to \mathbb{R}$ converges to $a \in \mathbb{R}$ if for every $\epsilon > 0$ there exists a $N \in \mathbb{N}$ such that for all $n > N$ distance between $f(n)$ and a is less than ϵ. Write "f converges to a" and "its negation" in terms of predicates and quantifiers. [4]

   Solution. f converges to a

   $\forall \epsilon > 0 \, \exists \, N \in \mathbb{N} \, \forall \, n \geq N(|f(n) - a| < \epsilon)$ [2]

   f not converges to a

   $\exists \epsilon > 0 \, \forall \, N \in \mathbb{N} \, \exists \, n \geq N(|f(n) - a| \geq \epsilon)$ [2]

3. Let $X = \mathbb{Z} \times \mathbb{Z}$ and $(A, B) \in X$. Define a relation R on X given by [4]

$$(m, n)R(u, v) \Leftrightarrow m - An = Bu - v.$$

Find all pairs $(A, B) \in X$ such the given relation is an equivalence relation. Determine the equivalence class of $(1, 2)$.

Solution. For reflexive:

$$(m, n)R(m, n) \Leftrightarrow m - An = Bm - n \Leftrightarrow m(1 - B) + n(1 - A) = 0$$

For the equation $m(1-B)+n(1-A) = 0$ to hold for all integers m and n, the coefficients must be zero, that is, $(A, B) = (1, 1)$. [1]

Thus $(m, n)R(u, v) \Leftrightarrow m - n = u - v$

R is symmetric as: If $m - n = u - v$, then $u - v = m - n$. [1]

R is transitive as: If $(m, n)R(u, v)$, i.e., $m-n = u-v$ and $(u, v)R(x, y)$, i.e., $u - v = x - y$, then $u - v = x - y$, i.e., $(m, n)R(x, y)$. [1]

The equivalevce class of $(1, 2)$ is:
$[(1, 2)] = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \,|\, m - n = -1\} = \{(m, m + 1) \,|\, m \in \mathbb{Z}\}$. [1]